Mediation Analysis

Ye Wang
University of North Carolina at Chapel Hill

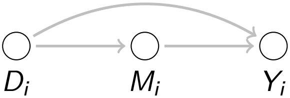
Linear Methods in Causal Inference
POLI784

Review

- In the previous class, we first reviewed methods that are valid under sequential ignorability, including trajectory balancing and IPW.
- We then investigated complexities caused by temporal interference in panel data.
- Methods based sequential ignorability can still work after we control for the treatment assignment history.
- But the outcome history now becomes a post-treatment variable and needs to be adjusted sequentially.
- If the data have a structure of staggered adoption, methods based on strict exogeneity are compatible with temporal interference.
- Otherwise, we have to decide which structural restrictions are more realistic.

Mediation

- Researchers are often interested in mechanisms underlying a causal relationship:



- Variables that stand for mechanisms are known as "mediators".
- Sailors know for a long time that eating fruits prevents you from getting scurvy.
- But only fresh fruits are effective as they contain vitamin C.
- Isolating such mechanisms is thus crucial for policy interventions.
- They may also deepen our understanding of the world.
- E.g., how does a message shown to the respondents change their opinion?
- Does it increase their knowledge, change their belief, or evoke certain emotions?

Define the mediation effect

- Consider a sample with N units, for each we observe Y_i, $D_i \in \{0, 1\}$, and a mediator M_i.
- The outcome Y_i is decided by both D_i and M_i: $Y_i = Y_i(D_i, M_i)$.
- Therefore multiple potential outcomes for each unit i:

$$Y_i = \begin{cases} Y_i(1, m), D_i = 1, M_i = m \\ Y_i(0, m), D_i = 0, M_i = m \\ Y_i(0, m'), D_i = 0, M_i = m' \\ Y_i(1, m'), D_i = 1, M_i = m'. \end{cases}$$

- The mediator's value is decided by D_i hence post-treatment:

$$M_i = \begin{cases} M_i(1), D_i = 1 \\ M_i(0), D_i = 0. \end{cases}$$

Define the mediation effect

- We can define the total effect for unit i as

$$\tau_{i,total} = Y_i(1, M_i(1)) - Y_i(0, M_i(0)).$$

- The natural mediation effect is

$$\tau_{i,nm}(d) = Y_i(d, M_i(1)) - Y_i(d, M_i(0)).$$

- The natural direct effect is

$$\tau_{i,nd}(d) = Y_i(1, M_i(d)) - Y_i(0, M_i(d)).$$

- We can see that $\tau_{i,total} = \tau_{i,nd}(d) + \tau_{i,nm}(1-d)$.

Define the mediation effect

- Recall our previous example where i stands for a country.
- D_i means whether country i has a high ethnic diversity; M_i indicates whether the country is developed; Y_i is the frequency of civil conflicts.
- The total effect captures the effect on civil conflicts generated by ethnic diversity through all possible channels.
- The mediation effect: the effect on civil conflicts when economic development changes from the level under control to the level under treatment, while ethnic diversity is fixed at d.
- Note that it differs from $Y_i(d, 1) - Y_i(d, 0)$.
- The direct effect: the effect of ethnic diversity on civil conflicts when development is fixed at the level under d.

Define the mediation effect

- The average total effect is $\tau = E[\tau_{i,total}]$, which equals the ATE.
- Similarly, the average natural direct effect is $\tau_{ANDE}(d) = E[\tau_{i,nd}(d)]$, and the average natural mediation effect is $\tau_{ANME}(d) = E[\tau_{i,nm}(d)]$.
- Imai, Keele, and Tingley (2010) call them "average direct effect" (ADE) and "average causal mediation effect" (ACME), respectively.
- The same decomposition holds

$$\tau = \tau_{ADE}(d) + \tau_{ACME}(1 - d).$$

Identify the mediation effect

- For simplicity, let's assume that D_i is randomly assigned:

$$D_i \perp \{Y_i(1, m), Y_i(0, m), M_i(0), M_i(1)\},$$
$$\varepsilon < P(D_i = 1) < 1 - \varepsilon.$$

- It is sufficient to identify the average total effect and the ATE on the mediator.
- To identify the ADE or ACME, we need to further assume that

$$M_i(d) \perp \{Y_i(1, m), Y_i(0, m)\} | D_i,$$
$$\varepsilon < P(M_i(d) = m) < 1 - \varepsilon.$$

- Imai, Keele, and Tingley (2010) call the two assumptions "sequential ignorability".
- It is different from what we saw in panel data analysis.

Identify the mediation effect

- We can easily estimate $E\left[Y_i(1, M_i(1))\right]$ and $E\left[Y_i(0, M_i(0))\right]$.
- Sequential ignorability allows us to estimate $E\left[Y_i(1, M_i(0))\right]$ and $E\left[Y_i(0, M_i(1))\right]$.
- Note that the assumption requires the manipulation of $M_i(d)$ rather than M_i.
- It cannot be simply guaranteed "by design" as the value of potential outcomes cannot be altered.
- Ideally, we need at least three parallel universes.
- In universe one, everyone is assigned with fresh oranges, and we observe $M_i(1)$ and $Y_i(1, M_i(1))$.
- In universe two, no one is assigned with fresh oranges, and we observe $M_i(0)$ and $Y_i(0, M_i(0))$.
- In universe three, everyone is assigned with fresh oranges, and we fix their level of vitamin C at $M_i(0)$ and observe $Y_i(1, M_i(0))$.
- The difference between universes one and three captures the ACME.

Estimate the mediation effect

- In reality, randomizing $M_i(0)$ or $M_i(1)$ is impossible as we do not know their values for everyone.
- We can only design experiments to identify the ACME indirectly under structural restrictions.
- Consider the parallel designs proposed by Imai et al. (2011).
- The idea is to estimate first τ and $\tau_{ADE}(d)$, and use their difference as an estimate of $\tau_{ACME}(1-d)$.
- We randomly divide the sample into two groups, G_1 and G_2.
- D_i is randomized in G_1, while both D_i and $M_i \in \{0, 1\}$ are randomized in G_2.
- We do not assume sequential ignorability.
- From G_1, we can estimate τ as before.

Estimate the mediation effect

- To estimate $\tau_{ADE}(0)$ or $\tau_{ADE}(1)$, we need a restriction that $Y_i(1,1) - Y_i(1,0) = Y_i(0,1) - Y_i(0,0)$ (no interaction).
- It implies that $\tau_{ADE}(0) = \tau_{ADE}(1) = \tau_{ADE}$.
- Define $p = P(D_i = 1)$ and $q = P(M_i = 1)$ in G_2, then

$$\begin{aligned}\hat{\tau}_{ADE} =& \frac{1}{N}\sum_{i=1}^{N}\frac{D_i M_i Y_i}{p} - \frac{1}{N}\sum_{i=1}^{N}\frac{(1-D_i)M_i Y_i}{p} \\ &+ \frac{1}{N}\sum_{i=1}^{N}\frac{D_i(1-M_i)Y_i}{1-p} - \frac{1}{N}\sum_{i=1}^{N}\frac{(1-D_i)(1-M_i)Y_i}{1-p}.\end{aligned}$$

- This estimator identifies

$$qE\left[Y_i(1,1) - Y_i(0,1)\right] + (1-q)E\left[Y_i(1,0) - Y_i(0,0)\right] = \tau_{ADE}.$$

Estimate the mediation effect

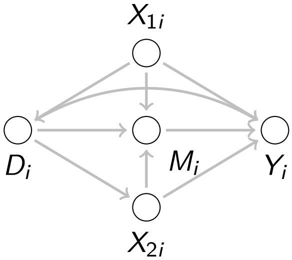
- Sequential ignorability is necessary in observational studies.
- The classical approach (Baron and Kenny 1986) is built upon the following linear models:

$$Y_i = \tau D_i + \beta M_i + \varepsilon_i,$$
$$M_i = \delta D_i + \nu_i.$$

- It assumes linearity, no interaction, and homogeneous treatment effects.
- Imai, Keele, and Yamamoto (2010) show that if all these restrictions are satisfied, then $\tau_{ACME} = \delta * \beta$ and $\tau_{ADE} = \tau$.
- It is straightforward to extend the first equation and assume $Y_i = \tau D_i + \beta M_i + \eta D_i * M_i + \varepsilon_i$.
- Then, $\tau_{ACME}(0) = \delta * \beta$, $\tau_{ACME}(1) = \delta * (\beta + \eta)$, $\tau_{ADE}(0) = \tau$, and $\tau_{ADE}(1) = \tau + \eta * \delta$.

Identify the mediation effect under strong ignorability

- Previous discussions have not accounted for the existence of confounders.
- We need to distinguish different types of confounders in mediation analysis.
- Consider the following graph:



- We can assume sequential ignorability conditional on X_{1i} but not X_{2i}.

Estimate the mediation effect under strong ignorability

- We can control for X_{1i} by adding extra terms into the linear equations.
- The modern approach is built upon nonparametric regression estimators.
- We need to estimate conditional expectations such as $\delta(D_i, M_i, X_{1i}) = E[Y_i|D_i, M_i, X_{1i}]$.
- Imai, Keele, and Tingley (2010) show that

$$\begin{aligned}&\tau_{ACME}(d)\\=&E\left[E_{M|D=1,X_1}[\delta(D_i, M_i, X_{1i})] - E_{M|D=0,X_1}[\delta(D_i, M_i, X_{1i})]\right]\end{aligned}$$

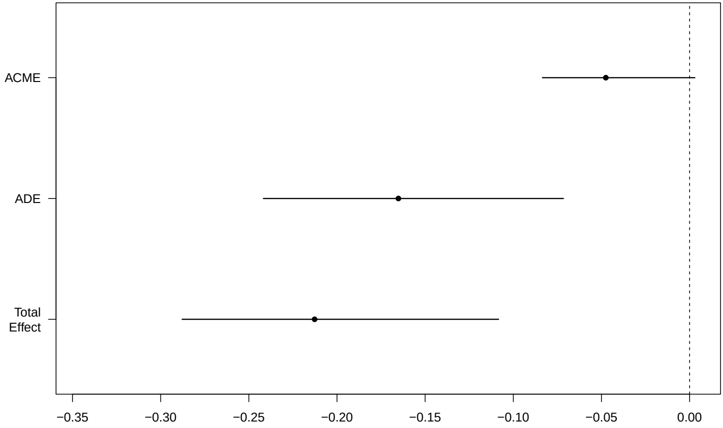
- They suggest an estimation algorithm based on Bayesian methods.
- This approach can be applied to continuous treatments or mediators.
- Sensitivity analysis is necessary to ensure that sequential ignorability holds.

Mediation analysis: application

- Consider the study in Lupu and Peisakhin (2017), which investigated the political legacy of Stalin's deportation of the Crimean Tatars.
- The authors conducted a survey on households with senior members who were born before the deportation.
- The treatment is whether any of the family members were victims of the deportation.
- The outcome is their support for Russia's annexation of Crimea.
- Mediators include multiple indicators about their identity and feelings over generations.
- We focus on the sub-sample of the third generation in these households.

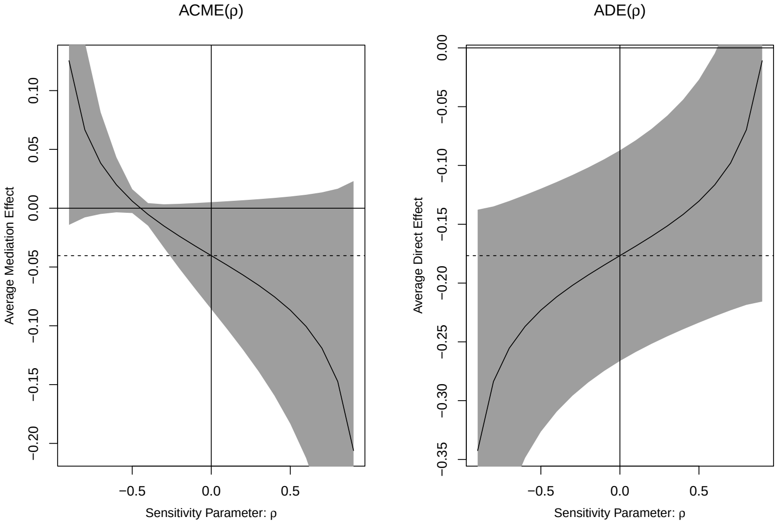
Mediation analysis: application

- We consider a single mediator, fear among the second generation in these households.



Mediation analysis: application

- The sensitivity analysis is built upon the linear model, and ρ is the correlation between ε_i and ν_i.



Estimate the mediation effect under strong ignorability

- What if we have confounders like X_{2i}?
- Acharya, Blackwell, and Sen (2016) show that we can identify a quantity known as the average controlled direct effect (ACDE):

$$\tau_{ACDE}(m) = E[Y_i(1, m) - Y_i(0, m)].$$

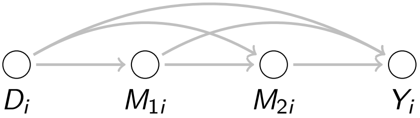
- We can replace the second part of sequential ignorability with

$$M_i \perp \{Y_i(1, m), Y_i(0, m)\} | D_i, X_{1i}, X_{2i}$$

- It is a familiar problem from panel data analysis.
- We need to account for the influence of the post-treatment variable X_{2i}.
- This can be done via IPW estimators.
- Or we can rely on regression models under structural restrictions.

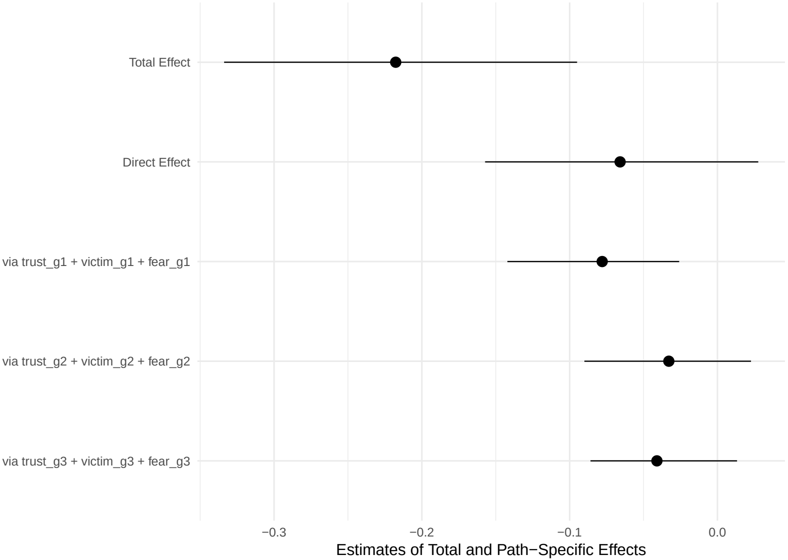
Ordered mediators

- Sometimes, there can be multiple mediators on the path from the treatment to the outcome.



- Mediators for the first generation affect mediators for the second generation.
- Zhou and Yamamoto (2020) show that we can similarly define the average causal mediation effect for each mediator.
- In the previous example, we can isolate the direct effect (D - Y), the effect through M_2 (D - M_2 - Y), and the effect through M_1 (D - M_1 - Y and D - M_1 - M_2 - Y).
- Identification requires that sequential ignorability holds for each mediator on the path.

Ordered mediators: application



Summary

- This course is dedicated to causal inference from the design-based perspective.
- Identifying causal relationships is impossible without assumptions.
- An identification assumption clarifies the source of randomness in treatment assignment
- A good research design ensures that the identification assumptions are likely satisfied.
- As a result, the estimates will be robust to structural restrictions imposed on the data generating process.
- Good designs should be justified by your understanding of theory and context.
- A bad design plus the abuse of statistical models often lead to empirical results that cannot be replicated or generalized.
- It is always necessary to test both the identification assumptions and the structural restrictions in your study.

Summary

- We studied a series of linear estimators over the semester, most of which have a regression representation.
- The critical question is whether the estimate converges to a quantity (estimand) that has a causal interpretation.
- We define causal estimands under the Neyman-Rubin framework.
- Each estimand is the average of the difference between two potential outcomes over a fixed population.
- Vanilla regression estimators may not converge to such averages of heterogeneous treatment effects.
- It is easier to fit a model than to figure out what quantity you are estimating.
- Takeaway: always try to understand the research question using the Neyman-Rubin framework before running any analysis!

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